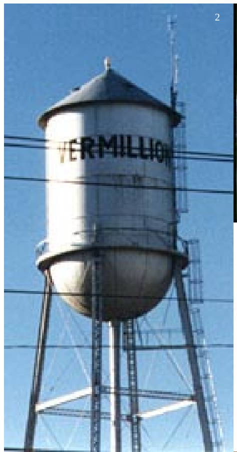
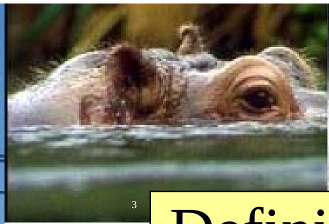
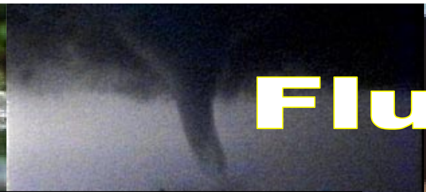
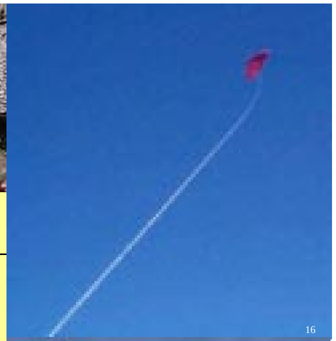
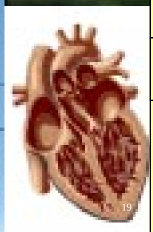
Fluids



Definitions

Fundamental Quantity:

A **fluid parcel** is a small bit of fluid with volume V, mass density ρ, and mass M.

$d_t = \partial_t + \vec{v}_i \partial_i$ — The **total time derivative** d_t

$f_i^{surface} = \partial_i T_{ij}$ — The **total stress tensor** T_{ij} producing the net surface force $f_i^{surface}$

$p_m = -\frac{1}{3} T_{ii}$ — The **mechanical pressure** p_m

$T_{ij} = -p_m \delta_{ij} + \tau_{ij}$ — The **viscous stress tensor** τ_{ij}

$dE = \rho\, e\, dV + \frac{1}{2} \rho\, v^2\, dV$ — The **internal energy** e

$\partial_t dQ = \partial_i q_i dV$ — The **heat flux rate** $\vec{q}(\vec{x}, t)$

$\partial_t dW = v_i f_i^{total} dV$ — The **work** dW done on the fluid

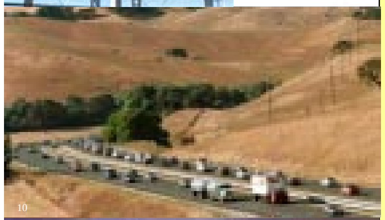
A **Newtonian fluid** is a fluid in which T_{ij} is a linear function of first spatial derivatives of velocity. In this case we can write T_{ij} in terms of η, the **dynamic viscosity**.

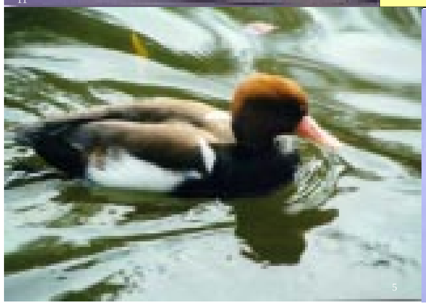
$\nu = \frac{\eta}{\rho}$ — The **specific viscosity** ν

An **incompressible fluid** satisfies $\partial_t \rho = 0$ and $\partial_i \rho = 0$

Steady flow satisfies $\partial_t \rho = 0$ and $\partial_t v_i = 0$

$\mathrm{Re} = \frac{\upsilon l \rho}{\eta}$ — The **Reynolds number** Re of a system with characteristic velocity υ, length l, density ρ, and viscosity η.





Observations

Conservation of mass: $d_t M = 0$

The forces on a fluid parcel may be written $\vec{f}^{total} = \vec{f}^{body} + \vec{f}^{surface}$

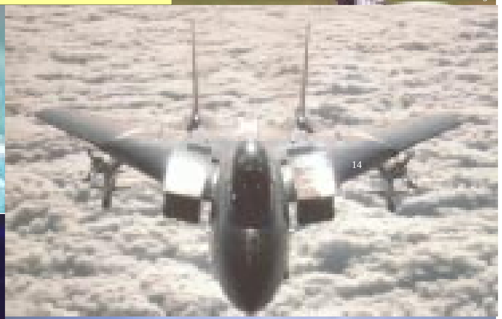
Conservation of momentum: $\rho\, d_t \vec{v} = \vec{f}^{total}$

Stokes' assumption: the mechanical pressure p_m equals the thermodynamic pressure p

Newton's law of cooling: for many systems, the relation $q_i = \kappa\, \partial_i T$ holds with suitable choice of the **thermal conductivity** κ

No-slip boundary condition: the appropriate fluid-solid boundary condition is typically $\vec{v}^{fluid} = \vec{v}^{solid}$





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