Computing project Part III

Assignment 8 − Symmetric Top with Torque

Add torque τ (divided by respective moments of inertia) to time derivatives (symbol N is not available in *Mathematica*).

```
ωdot[ω_] := {((I2 - I3) / I1) ω[[2]] ω[[3]], ((I3 - I1) / I2) ω[[1]] ω[[3]],
    ((I1 - I2) / I3) ω[[1]] ω[[2]]} + {τ[[1]] / I1, τ[[2]] / I2, τ[[3]] / I3};

imax = 1000;

t0 = 0;
dt = 0.01;
T = Table[t0, {i, 1, imax}];

I1 = 2;
I2 = 2;
I3 = 4;

EE = Table[0, {i, 1, imax}];
```

Set value for torque (the given values for the components have to be taken to be in the body frame, otherwise this problem cannot be solved without further information about the orientation of the rigid body in thhhe fixed frame):

```
τ = {0, 0, 4};
```

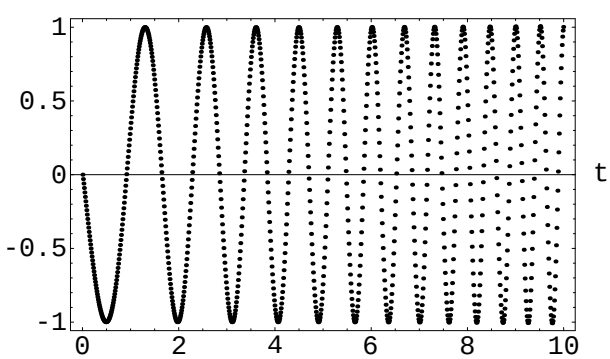
■ 1

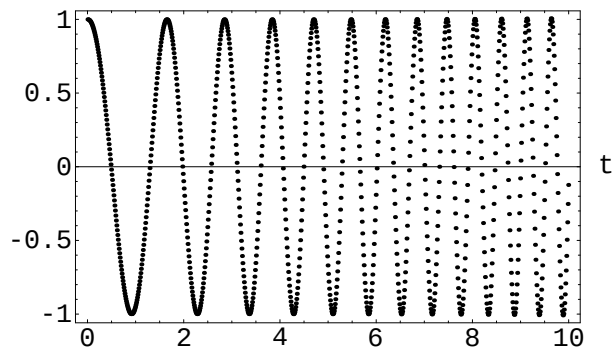
We expect the torque to drive the rigid body into rotation around the z axis.

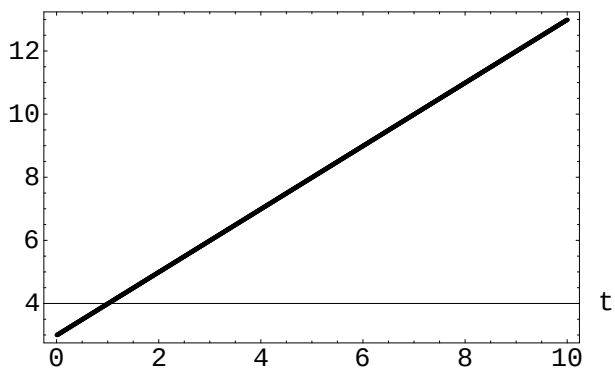
```
ω0 = {0, 1, 3};
ω = Table[ω0, {i, 1, imax}];

Do[ T[[i]] = T[[i - 1]] + dt;
 ωpred = ω[[i - 1]] + ωdot[ ω[[i - 1]] ] dt;
 ω[[i]] = ω[[i - 1]] + (ωdot[ ω[[i - 1]] ] + ωdot[ ωpred ]) dt / 2;
 , {i, 2, imax}]

EE = I1 Transpose[ω][[1]]^2 + I2 Transpose[ω][[2]]^2 + I3 Transpose[ω][[3]]^2;
```

Plot $\omega 1$, $\omega 2$ and $\omega 3$ as function of time. Plot energy as function of time.

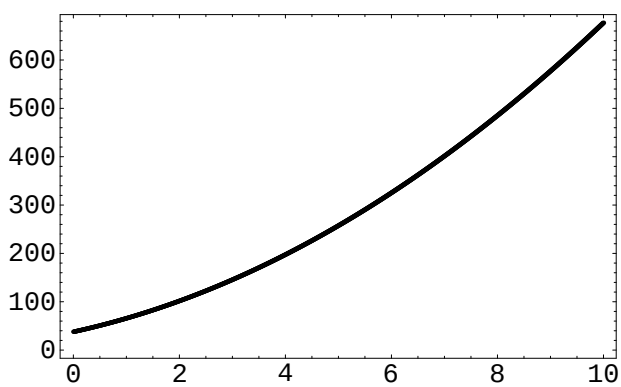
```
ListPlot[Transpose[{T, Transpose[ω][[1]]}], Frame → True, AxesLabel → {"t", "ω1"}];
ListPlot[Transpose[{T, Transpose[ω][[2]]}], Frame → True, AxesLabel → {"t", "ω1"}];
ListPlot[Transpose[{T, Transpose[ω][[3]]}], Frame → True, AxesLabel → {"t", "ω1"}];
```








```
ListPlot[Transpose[{T, EE}], Frame → True, AxesLabel → {"T", "Energy"}];
```



The $\omega 3$ component becomes the dominating component very fast. Energy is clearly not conserved.

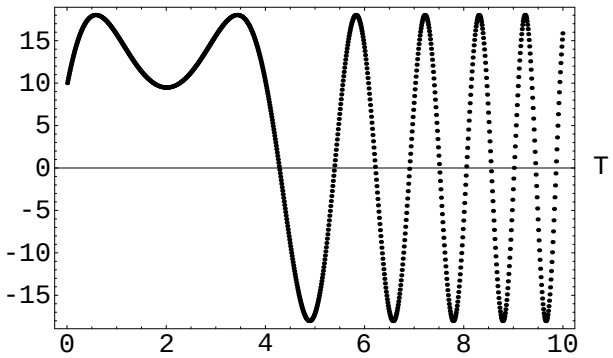
■ 2

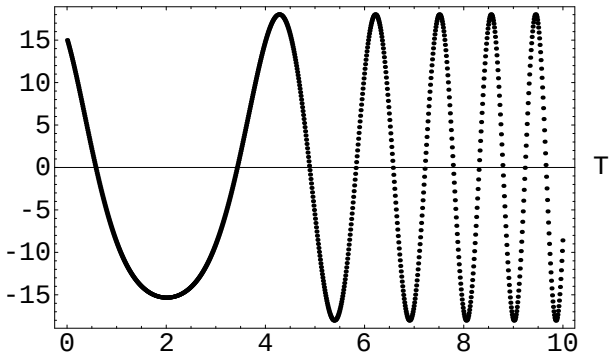
Start with $\omega 3$ being much smaller than $\omega 1$ and $\omega 2$.

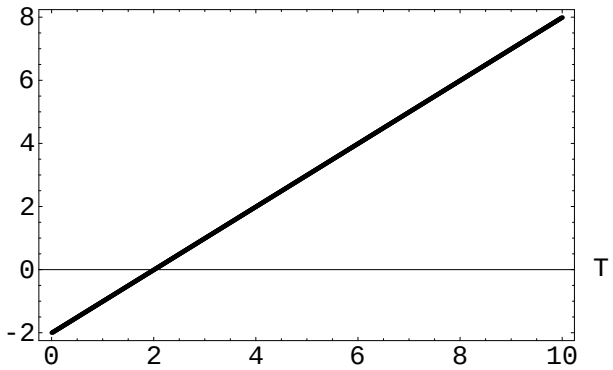
```
ω0 = {10, 15, -2};
ω = Table[ω0, {i, 1, imax}];
```

Calculate ω, energy and plot $\omega 1$, $\omega 2$, $\omega 3$ and energy.

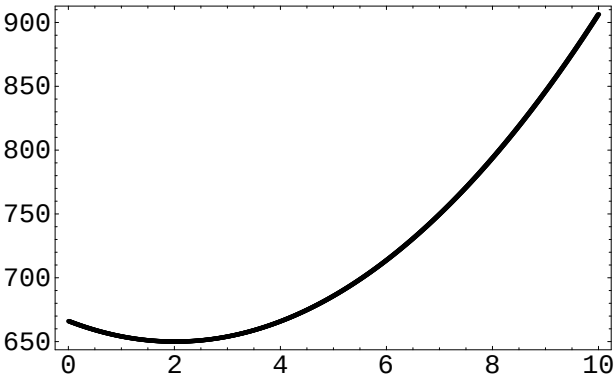
```
Do[ T[[i]] = T[[i - 1]] + dt;
 ωpred = ω[[i - 1]] + ωdot[ ω[[i - 1]] ] dt;
 ω[[i]] = ω[[i - 1]] + (ωdot[ ω[[i - 1]] ] + ωdot[ ωpred ]) dt / 2;
 , {i, 2, imax}]

EE = I1 Transpose[ω][[1]]^2 + I2 Transpose[ω][[2]]^2 + I3 Transpose[ω][[3]]^2;

ListPlot[Transpose[{T, Transpose[ω][[1]]}], Frame → True, AxesLabel → {"T", "ω1"}];
ListPlot[Transpose[{T, Transpose[ω][[2]]}], Frame → True, AxesLabel → {"T", "ω1"}];
ListPlot[Transpose[{T, Transpose[ω][[3]]}], Frame → True, AxesLabel → {"T", "ω1"}];
```








```
ListPlot[Transpose[{T, EE}], Frame → True, AxesLabel → {"T", "Energy"}];
```



It doesn’t matter what we choose at the beginning, $\omega 3$ will always dominate in the end.